

Assign-5

$$f(x, y) = e^{-(x+y)} \quad x > 0, y > 0$$

$$P(1 < x+y < 2)$$

$$P(x+y > 1)$$

$$P(0 < x+y < 2) - P(0 < x+y < 1)$$

$$\Rightarrow \int_0^{\infty} \int_0^{2-n} e^{-(x+y)} dx dy - \int_0^{\infty} \int_0^{1-n} e^{-(x+y)} dx dy$$

$$\Rightarrow \int_0^2 \int_y^2 e^{-x} dx dy - \int_0^1 \int_y^1 e^{-x} dx dy$$

$$\Rightarrow \int_0^2 -e^{-x} \Big|_y^2 - \int_0^1 -e^{-x} \Big|_y^1$$

$$\Rightarrow \int_0^2 e^{-y} - e^{-2} - \int_0^1 e^{-y} - e^{-1}$$

$$\Rightarrow -e^{-y} - e^{-2} + e^{-y} + e^{-1}$$

$$-e^{-y} - e^{-2} + e^{-y} + e^{-1} \Big|_0^2$$

$$\Rightarrow -e^{-2} - 2e^{-2} + 1 + e^{-1} + e^{-1} - 1$$

$$2e^{-1} - 3e^{-2}$$

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$$b \quad E(X/Y=10) = \int_{1/6}^1 \frac{x f(x,y)}{f_y(y)} dx$$

$$\Rightarrow \int_{1/6}^1 \frac{x \cdot 8xy}{4y(1-y^2)} dx = \frac{8y}{4y(1-y^2)} \left. \frac{x^2}{2} \right|_{1/6}^1$$

$$= \frac{2}{3(1-y^2)} \left[\frac{1}{2} - \frac{1}{216} \right] = \frac{430}{216(1-y^2)}$$

$$\Rightarrow \frac{430}{216 \times 3 \left(1 - \frac{1}{36}\right)}$$

$$= \frac{430 \times 12}{216 \times 3 \times \frac{35}{36}} = \frac{43}{63}$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = E(xy) - E(x) \cdot E(y)$$

$$\Rightarrow \int_0^1 \int_0^x xy \cdot 8xy \, dx \, dy$$

$$= \int_0^1 8x^2 y^2 \, dx \, dy = \frac{8}{3 \cdot 6} \frac{y^3}{3}$$

$$= \frac{8x^2 y^3}{3} \Big|_0^x = \frac{8x^5}{18} = \frac{4}{9}$$

$$\frac{8x^2}{3} x^3 = \frac{8x^5}{3}$$

$$E(n) = \int_0^1 n \cdot f_n(n) dn = 4/5$$

$$E(y) = \int_0^1 8ny dn = 4n^2 y \Big|_0^1$$

$$\int_0^1 \cancel{y \cdot 4y} dy = \cancel{4y^2 dy} = \frac{4y^3}{3} \Big|_0^1$$

$$\Rightarrow \frac{4}{3}$$

$$\int \cancel{8ny} \cdot y(f_n y)$$

$$\cancel{f_y = \int_0^n 8ny dy = \frac{8y^2}{2} n \Big|_0^n}$$

$$\Rightarrow \cancel{4n^2}$$

$$f_y = \int_y^1 8ny dy = 4n^2 y \Big|_y^1$$

$$4y[1-y^2]$$

$$E(y) = \int_0^1 y \cdot 4y[1-y^2] dy$$

$$\int 4y^2 - 4y^4 = \frac{4}{3} - \frac{4}{5} = \frac{20-12}{15} = \frac{8}{15}$$

$$\frac{x^3}{3} \Big|_0^1$$

$$\frac{n^2}{2}$$

$$= \frac{4}{9}$$

$$\frac{4}{9} - \frac{4}{5} \cdot \frac{8}{15} = \frac{20 - 32}{45}$$

$$\frac{19 \cdot 15 \times 5}{3 \cdot 5 \cdot 5}$$

$$= -\frac{12}{45}$$

$$\frac{4}{9} - \frac{32}{225} = \frac{100 - 32 \times 3}{225}$$

$$= \frac{4}{225}$$

$$\sigma \Rightarrow \frac{\frac{4}{225}}{\sqrt{\frac{4}{5} \times \frac{8}{15}}} = \text{var.} = E(n^2) - (E(n))^2$$

$$= 0.4923$$

(13) $f(n, y) = \frac{1}{4}(1 + ny) \quad |n| < 1$
 $|y| < 1$

$P(2n < y)$, ~~P(2n < y)~~

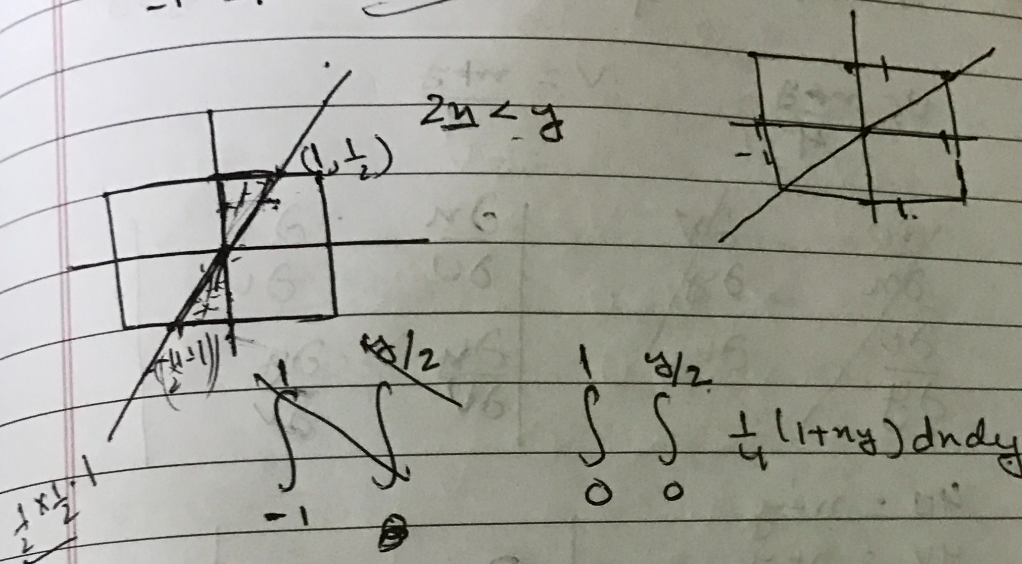
$$\int_{-1}^1 \int_{-y/2}^{y/2} \frac{1}{4}(1 + ny) \, dn \, dy$$

$$\Rightarrow \int_{-1}^1 \int_{-y/2}^{y/2} \frac{1}{4} \left(n + \frac{n^2}{2} y \right) \Big|_{-y/2}^{y/2} \, dy = \frac{1}{4} \left[\frac{1}{4} + \frac{1}{32} \right]$$

$$\frac{1}{4} \int_{-1}^1 \left(\frac{y}{2} + \frac{y^3}{8} \right) \, dy$$

$$\Rightarrow \frac{1}{4} \int_{-1}^1 y \, dy = \frac{1}{4} \left(\frac{y^2}{4} + \frac{y^4}{32} \right) \Big|_{-1}^1$$

$$\int_{-1}^1 \int_{-1}^{y/2} \frac{1}{4} (1+ny) \, dn \, dy$$



$$\int_0^1 \int_0^{y/2} \frac{1}{4} (1+ny) \, dn \, dy$$

$$\Rightarrow \int_0^1 dy \left[\frac{n}{4} + \frac{n^2 y}{8} \right] \Big|_0^{y/2}$$

$$= \int_0^1 dy \cdot \left[\frac{y}{8} + \frac{y^3}{32} \right]$$

$$\Rightarrow \int_0^1 \left(\frac{y}{8} + \frac{y^3}{32} \right) dy = \int_0^1 \frac{y^2}{16} + \frac{y^4}{32 \cdot 4}$$

$$\Rightarrow \left(\frac{1}{16} + \frac{1}{32 \cdot 4} \right) \times 2$$

$$\Rightarrow \frac{1}{8} + \frac{1}{64} = \frac{8+1}{64} = \frac{9}{64}$$

$$f(n, y) = \frac{1}{4} e^{-(n+y)/2} \quad \begin{matrix} n > 0 \\ y > 0 \end{matrix}$$

$$u = \frac{n+y}{2}$$

$$v = \frac{n+y}{2}$$

$\frac{\partial u}{\partial n}$	$\frac{\partial u}{\partial y}$	$\frac{\partial n}{\partial u}$	$\frac{\partial n}{\partial v}$
$\frac{\partial v}{\partial n}$	$\frac{\partial v}{\partial y}$	$\frac{\partial y}{\partial u}$	$\frac{\partial y}{\partial v}$

$$4u = n+y$$

$$4v = n+y$$

$$2(u+v) = n$$

$$2(u+v) = y$$

$$2(v-u) = y$$

$$= 4+4 = 8$$

$$v > 0$$

$$8 \cdot \frac{1}{4} e^{-\frac{4v}{2}} = 2e^{-2v}$$

$$\text{PMF} \quad 2e^{-2v} \quad (-\infty < v < \infty)$$

$$\int_{-\infty}^{\infty} 2e^{-2v} \cdot dv$$

$$\int_{-\infty}^{\infty} 2e^{-2v} dv$$

$$u+v > 0$$

$$u > -v \quad n > 0$$

$$v > 0$$

$$v < -u$$

$$v > u$$

$$v > u$$

$$\Rightarrow \frac{1}{2} e^{-2v} \Big|_{-\infty}^{\infty}$$

$$e^{-2v} \Big|_{-\infty}^{\infty}$$

$$e^{-2v} \Big|_{-\infty}^{\infty}$$

$$-e^{-2v} \Big|_{-\infty}^{-u}$$

$$\int_{-\infty}^{-u} 2e^{-2v} dv$$

$$\int_u^{\infty} 2e^{-2v} dv$$

$$\left[-e^{-2v} \right]_{-\infty}^{-u} = -\frac{1}{e^{2(-\infty)}} = -\frac{1}{e^{-\infty}}$$

$$u > -\infty$$

$$u > u$$

$$u < 0$$

$$u > 0$$

$$\begin{matrix} u > 0 \\ u < 0 \end{matrix}$$

$$f_x(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$$

$$f_y(y) = \begin{cases} 2e^{-2y} & y \geq 0 \\ 0 & y \leq 0 \end{cases}$$

$$u = x + y$$

$$u - y = x$$

$$v = y$$

$$u > 0$$

$$v > 0$$

$$x \geq 0$$

$$u - v > 0$$

$$u > v$$

$$u =$$

$$v \geq 0$$

$$u \geq v$$

$$0 \leq v \leq u$$

$$u \geq 0$$

$$Q \quad f(x, y) = 2xe^{-y} \quad 0 < x < 1, y > 0$$

$$= 0$$

$$x + y$$

$$\cancel{x + y = u}$$

$$\cancel{y = v}$$

$$\cancel{x = u - v}$$

$$\cancel{v > 0}$$

$$x = u$$

$$x + y = v$$

$$y = v - u$$

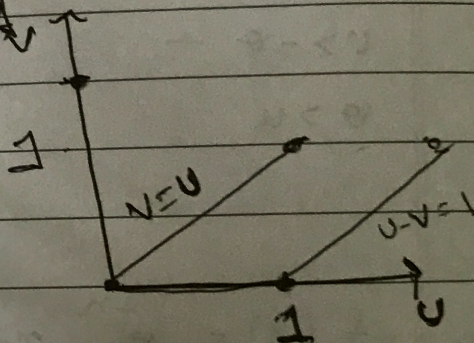
$$v > u$$

$$0 < u < 1$$

$$\cancel{0 < v < 1} \quad \boxed{v > 0}$$

$$0 < u - v < 1$$

$$1 < u < 2$$



Assignment-4

$$f_x(x) = \frac{2}{9}(x+1) \quad -1 < x < 2$$

$$y = x^2$$

$$x = \sqrt{y}$$

$$0 < x < 2$$

$$\left| \frac{dx}{dy} \right| = \frac{1}{2\sqrt{y}}$$

$$\frac{1}{2\sqrt{y}} \cdot \frac{2}{9}(\sqrt{y} + 1) \quad 0 < y < 4$$

when $-1 < x < 0$.

$$\frac{dx}{dy} = \frac{-1}{2\sqrt{y}}$$

$$x = -\sqrt{y}$$

$$\Rightarrow \frac{-1}{2\sqrt{y}} \cdot \frac{2}{9} (-\sqrt{y} + 1)$$

$$\Rightarrow \frac{1}{9\sqrt{y}} (-\sqrt{y} + 1) = \frac{1}{9} - \frac{1}{9\sqrt{y}}$$

$$= \frac{1}{9} - \frac{1}{9\sqrt{y}} \quad \text{for } 0 < y < 1$$

$$\Rightarrow \frac{2}{9}$$

we have to divide it into parts one is

$$= \frac{1}{2\sqrt{y}} \left[\frac{2}{9} (\sqrt{y} + 1) + (-\sqrt{y} + 1) \right]$$

symmetric

$$\Rightarrow \frac{2}{9 \times 2\sqrt{y}} \times 4 = \frac{2}{9\sqrt{y}}$$

either is increasing or decreasing

$$\text{for } -1 < x < 1, \quad 0 < y < 1$$

$$\frac{2}{9\sqrt{y}}$$

$$1 < y < 4$$

$$\frac{1}{9\sqrt{y}} (1 + \sqrt{y})$$

$$1 < y < 4$$

$$f(1-y) + f(y)$$

$$x = \frac{3}{2} - \sqrt{y}$$

$$\sqrt{y} = x - 3/2$$

Q2 $f_x(x) = \begin{cases} x/2 & 0 \leq x < 1 \\ 1/2 & 1 < x \leq 2 \\ (3-x)/2 & 2 < x \leq 3 \end{cases}$

$$\sqrt{y} = x - 3/2$$

$$y = \left(x - \frac{3}{2}\right)^2$$

$$x - \frac{3}{2} = \pm \sqrt{y}$$

$$x = +\sqrt{y} + \frac{3}{2}$$

$$x = -\sqrt{y} + \frac{3}{2}$$

$$\cancel{\sqrt{y}} \Rightarrow x = \frac{3}{2} + \sqrt{y}$$

$$\left| \frac{dy}{dx} \right| = \frac{1}{2\sqrt{y}} = \frac{1}{2} \left(\frac{3}{2} + \sqrt{y} \right) \times \frac{1}{2\sqrt{y}}$$

$$\Rightarrow \frac{1}{4\sqrt{y}} \left(\frac{3}{2} + \sqrt{y} \right) \quad \frac{1}{4} < y < \frac{9}{4}$$

Ans for $1 < x \leq 2$

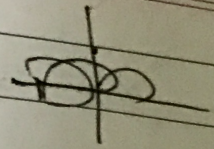
$$\Rightarrow \frac{1}{2\sqrt{y}} \left(\frac{1}{2} + \frac{1}{2} \right) \left(\frac{3}{2} + \sqrt{y} + \frac{3}{2} - \sqrt{y} \right)$$

$$\frac{1}{2\sqrt{y}} \Rightarrow \frac{1}{4\sqrt{y}} \Rightarrow 0 < y < \frac{1}{4}$$

Ans for $2 < x \leq 3$

$$\frac{1}{4\sqrt{y}} \left(3 - \frac{3}{2} - \sqrt{y} \right) = \frac{1}{4\sqrt{y}} \left(\frac{3}{2} - \sqrt{y} \right) \quad \frac{1}{4} < y < \frac{9}{4}$$

Q3 $f_x(n) = \frac{2n/\pi^2}{0} \quad 0 < x < \pi$

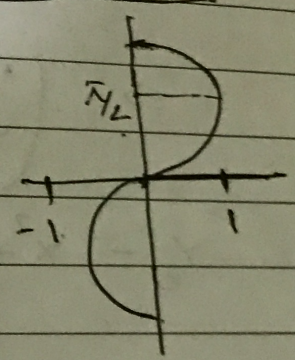


$x = \sin y$

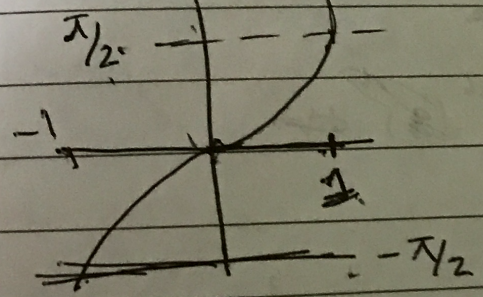
$x = \sin^{-1} y$

$-\pi/2 < y < \pi/2$

$\Rightarrow \frac{d \sin^{-1} y}{dy} = \frac{1}{\sqrt{1-y^2}}$



$x = \sin^{-1} y$



$-\pi/2 < y < \pi/2$

$0 < x < \pi$

$\Rightarrow \frac{1}{\sqrt{1-y^2}} \times \frac{\sin^{-1} y}{\pi^2}$

$0 < y < \pi/2$

$x \sim \text{Bin}(n, p)$

$y_1 = 3x + 4$

$n_c p^r q^{n-r}$

Q4. $Y_1 = 3X + 4$ Bin(n, p)

$$n C_x p^x q^{n-x}$$

$$\frac{y-4}{3} = x \quad n C_{\frac{y-4}{3}} p^{\frac{y-4}{3}} q^{n-\frac{y-4}{3}}$$

$$n C_{\frac{y-4}{3}}$$

$$\Rightarrow \left(n C_{\frac{y-4}{3}} \right) p^{\frac{y-4}{3}} (1-p)^{n-\frac{y-4}{3}}$$

Q5. $Y_3 = X^2 + 3$

$$16 C_0 \cdot (p)^{\frac{y-4}{3}} (1-p)^{n-\frac{y-4}{3}}$$

Q6. $e^{-2x} = e^x \cdot e^{-3x}$

$$P(X=1) = e^{-2} \cdot \frac{1}{2!}$$

$$x=3$$

$$P(X=2) = e^{-2} \cdot \frac{1}{2!}$$

$$\frac{3e^{-2} \cdot \frac{1}{4}}{3e^{-2} + \frac{1}{4}}$$

$$f(x) =$$

$$f(x) =$$

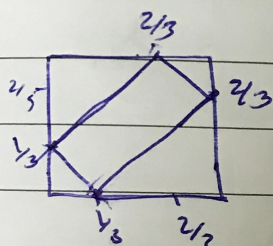
$$05' \quad \lambda = 2 \quad = e^{-2x} \frac{(2x)^n}{n!}$$

$$\Rightarrow \frac{e^{-2} \cdot 2^n}{n!} \quad \text{since } x=1$$

$$07' \quad \frac{1}{\lambda} = 300 \quad \lambda = \frac{1}{300} \quad P(X \leq 4)$$

$$\Rightarrow \frac{0.2}{100} \Delta$$

$$\Rightarrow e^{-6} + e^{-6} + e^{-6} \cdot \frac{6^2}{2!} + e^{-6} \cdot \frac{6^3}{3!}$$



$$\Rightarrow 2 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3}$$

$$1 - \left(\frac{1}{3} + \frac{1}{3} \right)$$

$$\frac{1}{x} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot x^2 = 1 - \frac{4}{9} = \frac{5}{9}$$

0)

09

$$f(x) = \frac{4}{5c} \quad \left(\frac{3}{4}c < x < 2c \right)$$

 \rightarrow cost ext

$$f(x) = \frac{1}{b-a}$$

$$E(x) = \frac{2c + \frac{3}{4}c}{2}$$

$$\Rightarrow \frac{1/c}{8}$$

$$f(x) = \frac{7}{5c}$$

$$f(1) = \frac{4}{5c}$$

det K.C. be luel

$$\text{Profit} = Kc - c$$

$$= c(K-1)$$

$$E(P) = c(K-1)f(c)$$

2c.

$$\int_{kc}^{2c} x(k-1) \cdot \frac{4}{5x} dx$$

$$(k-1)4 [2c - kc]$$

$$4(k-1)c(2-k) = 4c(2-k)(k-1)$$

$$\Rightarrow 4c(2k-2-k^2+k)$$

$$= 4c(-k^2+3k-2)$$

$$\Rightarrow 4c(-2k+3) = 0$$

$$k = \frac{3}{2}$$

Q10

$$f(t) = \lambda e^{-\lambda t}$$

$$\lambda = 50$$

$$\lambda = \frac{1}{50}$$

$$1 - (P(X=0) + P(X=1))$$

$$1 - \left(\frac{1}{50} + \frac{1}{50} e^{-\frac{1}{50}} \right)$$

Probability of bulb which will glow more than 100 hr

$$P(T > t) = e^{-\frac{1}{50} \times 100} = e^{-2} = P$$

$$1 - 10C_0 P^0 (1-e^{-2})^{10} + 10C_1 P^1 (1-e^{-2})^9$$

$$1 - (1-e^{-2})^{10} - 10e^{-2}(1-e^{-2})^9 = 0.400$$

Q11

Q12

Q13

$$\frac{1}{\lambda_1} = 5$$

$$\lambda_1 = \frac{1}{5}$$

B

$$\frac{1}{\lambda_2} = 2$$

$$\lambda_2 = \frac{1}{2}$$

$$P(B) = \frac{3}{4}$$

$$P(A) = \frac{1}{4}$$

$$\frac{3}{4} \cdot e^{-3/2} + \frac{1}{4} e^{-1/5} = \frac{3}{4} e^{-3/2} + \frac{1}{4} e^{-1}$$

$$P(T > t)$$

$$R(t) = \prod_{i=1}^n R_i(t)$$

$$\Rightarrow \cancel{e^{-\lambda t}} = e^{-\sum_{i=1}^n \lambda_i t}$$

$$\Rightarrow 1 - e^{-\sum_{i=1}^n \lambda_i t}$$

Q3

$$\frac{\gamma}{\lambda} = 20 \quad \sqrt{\gamma} \quad \frac{\sqrt{\gamma}}{\lambda} = 10$$

$$\Rightarrow \begin{aligned} \sqrt{\gamma} &= 2 \\ \gamma &= 4 \end{aligned}$$

$$\lambda = 0.2$$

$$P(T_\gamma > t) = \sum_{j=0}^{4-1} \frac{e^{-\lambda t} (\lambda t)^j}{j!}$$

$$\Rightarrow \sum_{j=0}^3 \frac{e^{-3} (3)^j}{j!}$$

within 15 days

$$= 1 - P(T_r > 15)$$

Q.14 $f_n(x) = \alpha p \otimes x \otimes e^{p(1-\alpha n^p)}$

$p \otimes \frac{1 - e^{-\alpha \cdot 100^2}}{e^{-\alpha \cdot 90^2}} = 0.15$

$\frac{1 - e^{-\alpha \cdot 10000}}{e^{-8100\alpha}} = 0.15$

$1 - e^{-10000\alpha} = 0.15 e^{-8100\alpha}$

$1 = 0.15 e^{-8100\alpha} + e^{-10000\alpha}$

$\Rightarrow \frac{1}{0.15} e^{8100\alpha} = e^{18100\alpha} = 1$

Q.15 $P(90 < n < 100)$

$\Rightarrow \frac{e^{-\alpha \cdot 90^2} - e^{-\alpha \cdot 100^2}}{e^{-\alpha \cdot 90^2}}$

$\Rightarrow \frac{e^{-8100\alpha} - e^{-10000\alpha}}{e^{-8100\alpha}}$

$\Rightarrow 1 - e^{-1900\alpha} = 0.15$

$0.85 = e^{-1900\alpha}$

$\alpha = 10.8555$

$\alpha = \frac{30}{2.9}$

$$P(n < 45) = 0.10$$

$$P(45 < n < 60)$$

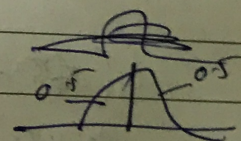
$$P(60 < n < 75)$$

$$P(n > 75) = 0.5$$

$$P\left(-\infty < n < \frac{45-4}{\sigma}\right) = 0.10$$

$$P\left(\frac{75-4}{\sigma} < X < \infty\right) = 0.05$$

$$\therefore P\left(0 < Z < \frac{45-4}{\sigma}\right) = 0.40$$



$$\Phi\left(\frac{45-4}{\sigma}\right) - \Phi(0) = 0.4$$

$$\Phi\left(\frac{45-4}{\sigma}\right) = 0.9$$

$$\Phi\left(\frac{75-4}{\sigma}\right) = 0.95$$

$$\frac{45-4}{\sigma} = 1.28$$

$$\frac{75-4}{\sigma} = 1.66$$

$$\frac{45-4}{\sigma} = -1.28$$

$$4 = 1.28\sigma + 75$$

$$\frac{4-45}{\sigma} = 1.28$$

$$\frac{75-4}{\sigma} = 1.66$$

$$\begin{aligned} 75 - 1.28\sigma \\ - 45 \\ = 1.66\sigma \end{aligned}$$

$$30 = 1.66\sigma - 1.28\sigma$$

$$30 = 1.94\sigma$$

$$\sigma = \frac{30}{1.94} = 15.46$$

$$\mu = 58.06$$

$$\phi(45 < X < 60)$$

$$\Rightarrow \phi\left(\frac{45 - 58.06}{10.20} < X < \frac{60 - 58.06}{10.20}\right)$$

$$\Rightarrow \phi(-1.28)$$

$$\phi(1.9) - \phi(-1.28)$$

$$0.5753 - 0.1003 = 0.475$$

$$\phi(60 < X < 75)$$

$$\phi(1.66) - \phi(1.99)$$

$$0.9505 - 0.5753 = 0.3752$$

Q6

$$\mu = 3$$

$$\sigma = 0.005$$

$$d_1 = 3 - 0.01 = 2.99$$

$$d_2 = 3 + 0.01 = 3.01$$

$$\phi\left(\frac{2.99 - 3}{0.005} < X < \frac{3.01 - 3}{0.005}\right)$$

$$\Rightarrow \phi(-2) < X < 2$$

$$\phi(2) - \phi(-2)$$

$$2\phi(2) = 1 -$$

$$\mu = 1.9744$$

$$0.9972 - 0.0228$$

$$\Rightarrow 0.9744$$

Q17

$$\mu = 0.9, \sigma = 0.003$$

$$P\left(\frac{0.9 - 0.005}{0.003} < Z < \frac{0.9 + 0.005}{0.003}\right)$$

$$P\left(\frac{-0.895}{0.003} < Z < \frac{0.905}{0.003}\right)$$

$$\therefore P(-1.66 < Z < 1.66) = \Phi(1.66) - \Phi(-1.66)$$

$$\Rightarrow 2\Phi(1.66) - 1$$

$$= 0.901$$

$$\therefore \% \text{ of defective} = 1 - 0.901$$

$$= 0.099$$

$$P\left(\frac{0.895}{\sigma} < Z < \frac{0.905}{\sigma}\right) = 0.99$$

$$\Rightarrow \cancel{\Phi\left(\frac{0.905}{\sigma}\right) - \Phi\left(\frac{0.895}{\sigma}\right)} = \cancel{\Phi(2.33)}$$

$$\Rightarrow \frac{0.90}{\sigma} = 2.33$$

$$\sigma = \frac{0.10}{2.33}$$

$$\cancel{2\Phi\left(\frac{0.905}{\sigma}\right) - \Phi\left(\frac{0.895}{\sigma}\right)} = \cancel{\Phi(2.33)}$$

$$\Rightarrow 2\Phi(2.33) - 1 = \Phi(2.33)$$

Q11

classmate

Date

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Q18

$$\mu = 2$$

$$\sigma = 10$$

$$P(A)$$

$$P\left(\frac{X+4}{\sigma}\right) = 0.05$$

$$\Phi\left(\frac{X+2}{10}\right) = 0$$

$$\Phi\left(\frac{200-X}{10}\right) = 0.95$$

$$\frac{200-X}{10} = 1.66$$

$$200 - X = 16.6$$

$$X = 183.4$$

$$\frac{X-\mu}{\sigma} = -1.28$$

$$P(Z > 0.1)$$

$$\frac{X-200}{10} = -1.28$$

$$X = 200 - 12.8 = 187.2$$

Q19

$$\mu = 74$$

$$\sigma^2 = 62.41$$

$$\sigma = 7.9$$

$$0 < X < 4$$

$$P\left(\frac{X-\mu}{\sigma}\right)$$

$$P\left(Z < \frac{4-\mu}{\sigma}\right)$$

$f(t) = \frac{\lambda t^{\lambda-1} e^{-\lambda t}}{\Gamma(\lambda)}$ → To find Probability of this we need to integrate it.

$$\int \frac{\lambda t^{\lambda-1} e^{-\lambda t}}{\Gamma(\lambda)}$$

$$z(t) = 0.027 + 0.00025(t-40)^2$$

$$Q_R = e^{-\alpha n \beta} \quad H = \alpha \beta e^{-\alpha n \beta}$$

$$R(t) = e^{-\int_{40}^t z(s) ds}$$

$$\alpha = 0.006 \quad \beta = 0.5$$

$$R(t) = e^{-\alpha t \beta}$$

$$e^{-\lambda t} \Rightarrow e^{-\frac{1}{25000} t}$$

$$\Rightarrow e^{-\alpha t \beta} \cdot e^{-\frac{t}{\lambda}}$$

$$e^{-\alpha t(1-R(t)) + \alpha t}$$

$$P(T < 2000)$$

A
B

$$e^{-\alpha t} (1 - R(t)) + (1 - e^{-\alpha t}) R(t)$$

$$\neq e^{-\alpha t} R(t)$$

$$e^{-\alpha t} - e^{-\alpha t} R(t) + R(t) - e^{-\alpha t} R(t)$$

$$+ e^{-\alpha t} R(t)$$

$$\text{or } P(T \leq t) = 1 - e^{-\lambda t}$$

$$\boxed{P(n \geq 2)}$$

$$\mu = 160, \sigma$$

$$P(120 < X < 200) = 0.80$$

ϕ

$$\Rightarrow \phi\left(\frac{200 - \mu}{\sigma}\right)$$

$$P\left(-\frac{40}{\sigma} < X < \frac{40}{\sigma}\right) = 0.80$$

$$\phi(n) + \phi(-n) = 1$$

$$\Rightarrow \phi\left(\frac{40}{\sigma}\right) - \phi\left(-\frac{40}{\sigma}\right) = 0.80$$

$$\Rightarrow 2\phi\left(\frac{40}{\sigma}\right) - 1 = 0.80$$

$$2\phi\left(\frac{40}{\sigma}\right) - 1 = 0.80$$

$$P(X \geq 170) / P(X \geq 140) \Rightarrow \frac{P\left(\frac{160 - 170}{\sigma}\right)}{P\left(\frac{160 - 140}{\sigma}\right)}$$

$$\Rightarrow \Rightarrow \frac{10}{30}$$

Q21

$$C_0 \cdot P(6 < X < 8) - C_1 P(X < 6) - C_2 P(X > 8)$$

$$\Rightarrow C_0 \left(\frac{8 - \mu}{\sigma} < X < \frac{8 - \mu}{\sigma} \right) - C_1 \left(\frac{6 - \mu}{\sigma} \right) - \frac{C_2}{2} \left(1 - \phi\left(\frac{8 - \mu}{\sigma}\right) \right)$$

$$C_0 (\phi(8 - \mu) - \phi(6 - \mu)) - C_1 \phi(6 - \mu)$$

$$- C_2 + C_2 \phi(8 - \mu)$$

$$c_0 [\phi(8-4) - \phi(6-4)] - c_1 \phi(6-4) + c_2 \phi(8-4)$$

~~$$c_0 e^{-\frac{(8-4)^2}{2}} - c_1 e^{-\frac{(6-4)^2}{2}} + c_2 e^{-\frac{(8-4)^2}{2}}$$~~

$$c_0 (e^{-\frac{(8-4)^2}{2}} - e^{-\frac{(6-4)^2}{2}}) - c_1 e^{-\frac{(6-4)^2}{2}} + c_2 e^{-\frac{(8-4)^2}{2}} = 0$$

$$\phi(8-4) (c_0 + c_2) - \phi(6-4) (c_0 + c_1) = 0$$

~~$$\Rightarrow c_0 e^{-\frac{(8-4)^2}{2}} [1 - e^{-\frac{(6-4)^2 - (8-4)^2}{2}}] = c_1 e^{-\frac{(6-4)^2}{2}} + c_2 e^{-\frac{(8-4)^2}{2}}$$~~

$$e^{-\frac{(8-4)^2}{2}} [c_0 + c_2] = (c_1 + c_0) e^{-\frac{(6-4)^2}{2}}$$

$$\frac{e^{-\frac{(8-4)^2}{2}}}{e^{-\frac{(6-4)^2}{2}}} = \frac{c_1 + c_0}{c_0 + c_2}$$

$$\Rightarrow -2 \frac{(8-4)^2}{2} \text{ taking } \ln$$

~~$$\Rightarrow -2 \frac{(8-4)^2}{2} + 2 \frac{(6-4)^2}{2} = \frac{c_1 + c_0}{c_0 + c_2}$$~~

~~$$-8 + 4 + 6 - 4$$~~

$$\ln e^{-\left(\frac{8-y}{\sqrt{2}}\right)^2} - \ln e^{-\left(\frac{6-y}{\sqrt{2}}\right)^2}$$

$$\Rightarrow -2\left(\frac{8-y}{\sqrt{2}}\right) + 2\left(\frac{6-y}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{16+2y}{\sqrt{2}} + \frac{12-2y}{\sqrt{2}}$$

$$\Rightarrow -\frac{4}{\sqrt{2}}$$

$$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$$

$$= \ln e^{-\left(\frac{8-y}{\sqrt{2}}\right)^2} - \ln e^{-\left(\frac{6-y}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \cancel{-2} - \left(\frac{8-y}{\sqrt{2}}\right)^2 + \left(\frac{6-y}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{-\left(8^2 + y^2 - 16y\right) + \left(36 + y^2 - 12y\right)}{\sqrt{2}}$$

$$\Rightarrow \frac{-64 - y^2 + 16y + 36 + y^2 - 12y}{\sqrt{2}}$$

$$\frac{-64}{28}$$

$$\Rightarrow \frac{-28 + 4y}{\sqrt{2}} = -\frac{7+4y}{\sqrt{2}}$$

$$\frac{-7+4y}{\sqrt{2}} = -14 + 2y =$$

$$\left(\frac{6-4}{\sqrt{2}}\right)^2 - \left(\frac{8-4}{\sqrt{2}}\right)^2$$

$$\frac{(6-4)^2 - (8-4)^2}{2} = \frac{36+4^2-12\cdot 4 - (64+4^2-16\cdot 4)}{2}$$

$$\Rightarrow \frac{36+4^2-12\cdot 4 - 64-4^2+16\cdot 4}{2}$$

$$\Rightarrow \frac{-28+4\cdot 4}{2} =$$

$$-14+2\cdot 4 = \frac{C_0+C_1}{C_2+C_0}$$

$$P'(Z \leq \ln(\frac{x-0.8}{0.1}))$$

$$1 - \phi(\ln(\frac{2.7-0.8}{0.1}))$$

$$\Rightarrow 1 - \ln(\frac{1.9}{0.1})$$

$$1 - \ln 19$$

$$1 - 2.944$$

$$\Rightarrow 1 - \phi\left(\frac{\ln(2.7)-4}{0}\right)$$

$$\Rightarrow 1 - \phi\left(\frac{0.993-0.8}{0.1}\right)$$

X is 0

$$x = e^x$$

$$1 - \phi\left(\frac{0.993}{0.1}\right)$$

$$1 - \phi(0.193)$$

$$1 - 0.9732$$

$$\left(\frac{z-4}{\sigma}\right) \left(\frac{z+4}{\sigma}\right) Y = \ln x$$

$$\frac{4-2}{\sigma}$$

$$\frac{4-2}{\sigma}$$

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$$0.8 - 2.71$$

$$0.1$$

$$P(Y < \ln x < z) = 0.95$$

$$P\left(\frac{4-4}{\sigma} < z < \frac{4+4}{\sigma}\right) = 0.95$$

$$= 2\phi\left(\frac{\ln(z)-4}{\sigma}\right) - 1 = 0.95$$

$$= 2\phi\left(\frac{\ln(z)-4}{\sigma}\right) = 1.95$$

$$\phi\left(\frac{\ln(z)-4}{\sigma}\right) = 1.95/2$$

$$\phi\left(\frac{\ln(z)-0.8}{0.1}\right) = 1.95/2$$

$$\frac{\ln(z)-0.8}{0.1} = 1.97$$

$$\ln z - 0.8 = 0.197$$

$$\ln z = 0.997$$

$$z = 2.71$$

$$\frac{2.71-0.8}{0.1}$$

$$\Rightarrow \frac{1.91}{0.1}$$

\Rightarrow

$$\frac{0.8-0.99}{0.1}$$

$$\frac{0.8 - \ln z}{0.1} = 0.1.97$$

$$0.603 = \ln(z)$$

8-271

0.1

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$$\textcircled{01} \quad U = |u| \quad \& \quad V = y^2$$

$$0 \leq U \leq 1 \quad 1 \leq V \leq 4$$

$$P(U=0 | V=1) = \frac{1}{12}$$

$$P(U=0 | V=4) = \frac{1}{12}$$

$$P(U=1 | V=1) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$P(U=1 | V=4) = \frac{1}{6} + \frac{1}{6} + \frac{1}{12} + \frac{1}{12}$$

$$\Rightarrow \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{1}{2}$$

$V \backslash U$	0	1
1	$\frac{1}{12}$	$\frac{1}{3}$
4	$\frac{1}{12}$	$\frac{1}{2}$

$$\textcircled{02} \quad Z^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$\Rightarrow \frac{X^2 + Y^2}{2}$$

$$X = x_1 - x_2$$

$$Y = y_1 - y_2$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}}$$

$$x_1 - x_2 \sim N(0, 1)$$

$$y_1 - y_2 \sim N(0, 1)$$

Q3.

$$x_1 = \lambda e^{-\lambda x_1}$$

$$x_2 = \lambda e^{-\lambda x_2}$$

$$x_1 = x_1/x_2 = \frac{\lambda e^{-\lambda x_1}}{\lambda e^{-\lambda x_2}}$$

$$y = e^{-\lambda(x_2 - x_1)}$$

$$y_1 = x_1/x_2$$

$$y_2 = x_1 + x_2$$

$$x_2 = x_1 x_2 + x_2$$

$$x_2 = \frac{y_2}{1 + y_1}$$

$$x_1 = \frac{y_1 y_2}{1 + y_1}$$

$$\begin{vmatrix} \frac{dx_1}{dy_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{dx_2}{dy_1} & \frac{dx_2}{dy_2} \end{vmatrix} = \begin{vmatrix} \frac{y_2}{1+y_1} & \frac{y_1}{1+y_1} \\ -\frac{y_2}{(1+y_1)^2} & \frac{1}{1+y_1} \end{vmatrix}$$

$$= \frac{y_2}{(1+y_1)^2} + \frac{y_1 y_2}{(1+y_1)^3}$$

$$\Rightarrow \begin{vmatrix} -\frac{y_2 y_1}{(1+y_1)^2} + \frac{y_2}{(1+y_1)} & \frac{y_1}{1+y_1} \\ -\frac{y_2}{(1+y_1)^2} & \frac{1}{1+y_1} \end{vmatrix}$$

$$\Rightarrow \frac{-y_2 y_1}{(1+y_1)^3} + \frac{y_2}{(1+y_1)} + \frac{y_1 y_2}{(1+y_1)^3}$$

$$\Rightarrow \frac{y_2}{(1+y_1)^2}$$

$$f(y_1, y_2) = \lambda^2 e^{-\lambda(y_1 + y_2)} \cdot \left[\frac{y_2}{(1+y_1)^2} \right]$$

⇒

$$f(y_1, y_2) = \frac{\lambda^2 e^{-\lambda(y_1 + y_2)} \cdot y_2}{(1+y_1)^2}$$

$$f(y_1, y_2) = \frac{\lambda^2 e^{-\lambda y_2} y_2}{(1+y_1)^2} \quad \text{--- Joint}$$

$$f_{y_1} \Rightarrow \int_0^{\infty} \frac{\lambda^2 e^{-\lambda y_2} y_2}{(1+y_1)^2} dy_2$$

$$\Rightarrow \frac{\lambda^2}{1+y_1^2} \int_0^{\infty} y_2 e^{-\lambda y_2} dy_2$$

$$= \frac{\lambda^2}{1+y_1^2} \left[\frac{y_2 e^{-\lambda y_2}}{\lambda} - \int \frac{-e^{-\lambda y_2}}{\lambda} \right]$$

$$\frac{\lambda^2}{1+y_1^2} \left[\frac{y_2 e^{-\lambda y_2}}{\lambda} - \frac{e^{-\lambda y_2}}{\lambda^2} \right]_0^{\infty}$$

$$\frac{\lambda^2}{1+y_1^2} \left[0 - 0 - \left(0 - \frac{1}{\lambda^2} \right) \right] = \frac{1}{(1+y_1)^2}$$

$$\text{Qy } x_1 = x_1^2 + x_2^2 \quad y_2 = x_1/x_2 \quad y_1 > 0$$

$$-\infty < y_2 < \infty$$

$$F \quad \phi = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$\phi(n_1) = \frac{1}{\sqrt{2\pi}} e^{-n_1^2/2} \quad \phi(n_2) = \frac{1}{\sqrt{2\pi}} e^{-n_2^2/2}$$

$$\Rightarrow x_1 = x_2^2 + n_2^2$$

$$y_1 = x_2^2 (1 + y_2^2)$$

$$x_2^2 = \frac{y_1}{1 + y_2^2}$$

$$\therefore x_2 = \left(\frac{y_1}{1 + y_2^2} \right)^{1/2}$$

$$x_1 = x_2^2 + \frac{y_1}{1 + y_2^2}$$

$$\frac{y_1 - \frac{y_1}{1 + y_2^2}}{1 + y_2^2} = x_1^2 \Rightarrow \frac{y_1 + y_1 y_2^2 - y_1}{1 + y_2^2} = x_1^2$$

$$\therefore x_1^2 = \frac{+ y_1 y_2^2}{1 + y_2^2}$$

$$\therefore x_1 = \left(\frac{+ y_1 y_2^2}{1 + y_2^2} \right)^{1/2}$$

$$x_2 = \left(\frac{y_1}{1+y_2^2} \right)^{1/2}$$

$$x_1 = \left(\frac{y_1 y_2^2}{1+y_2^2} \right)^{1/2}$$

$$\begin{vmatrix} \frac{dx_1}{dy_1} & \frac{dx_1}{dy_2} \\ \frac{dx_2}{dy_1} & \frac{dx_2}{dy_2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \frac{y_2}{\sqrt{1+y_2^2}} & \frac{1}{2\sqrt{y_1}} \\ \frac{y_2^2}{\sqrt{1+y_2^2}} & \frac{1}{2\sqrt{y_1}} \end{vmatrix}$$

$$\sqrt{y_1} \left[\frac{1}{2\sqrt{y_1}} \cdot \frac{y_2^2}{1+y_2^2} - \frac{1}{2\sqrt{y_1}} \cdot \frac{y_2}{1+y_2^2} \right]$$

$$J = -\frac{1}{2(1+y_2^2)^{3/2}}$$

$$f(y_1, y_2) = \frac{1}{2\pi} e^{-\frac{y_1}{2}} \times \frac{1}{2(1+y_2^2)^{3/2}}$$

$$x_2 = \sqrt{\frac{y_1}{1+y_2^2}}$$

$$x_1 = \sqrt{\frac{y_1 y_2^2}{1+y_2^2}}$$

$$\frac{(1+y_2^2)^{-\frac{1}{2}}}{\frac{1}{2}}$$

$$x_2 = -\sqrt{\frac{y_1}{1+y_2^2}}$$

$$x_1 = -y_2 \sqrt{\frac{y_1}{1+y_2^2}}$$

$$\frac{dx_1}{dy_1}$$

$$\frac{dx_1}{dy_2}$$

$$\frac{dx_1}{dy_1} = \frac{-y_2}{\sqrt{1+y_2^2}} \cdot \frac{1}{2\sqrt{y_1}}$$

$$\frac{dx_2}{dy_1}$$

$$\frac{dx_2}{dy_2}$$

$$\Rightarrow -y_2$$

$$2\sqrt{y_1(1+y_2^2)}$$

$$\frac{dx_1}{dy_2} = -\sqrt{y_1} \left[\frac{y_2 \cdot (1+y_2^2)^{\frac{1}{2}}}{\frac{1}{2}} \cdot 2y_2 - \frac{1}{\sqrt{1+y_2^2}} \right]$$

$$\Rightarrow -\sqrt{y_1} \left[2y_2^2 (1+y_2^2)^{\frac{1}{2}} - \frac{1}{\sqrt{1+y_2^2}} \right]$$

$$\frac{dx_2}{dy_1} = \frac{-1}{\sqrt{1+y_2^2}} \cdot \frac{1}{2\sqrt{y_1}} = \frac{-1}{2\sqrt{y_1(1+y_2^2)}}$$

$$\frac{dx_2}{dy_2} = -\sqrt{y_1} \sqrt{1+y_2^2}$$

$$\frac{(1+y_2^2)^{-1/2+1}}{\frac{1}{2} \cdot 2y_2}$$

$$\frac{dn_1}{dy_2} = -\sqrt{y_1} \left[\frac{y_2 \cdot 2y_2}{(1+y_2^2)^{3/2}} - \frac{1}{\sqrt{1+y_2^2}} \right]$$

$$= -\sqrt{y_1} \left[\frac{2y_2^2}{(1+y_2^2)^{3/2}} - \frac{1}{\sqrt{1+y_2^2}} \right]$$

$$\frac{dn_2}{dy_2} = -\sqrt{y_1} (1+y_2^2)^{-3/2} \cdot 2y_2$$

$$\Rightarrow \frac{y_2^2 \sqrt{y_1}}{2 \sqrt{y_1} (1+y_2^2)^2} = \frac{y_2^2}{2(1+y_2^2)^2}$$

$$\frac{-y_2^2}{(1+y_2^2)^2} + \frac{1}{2\sqrt{y_1} (1+y_2^2)}$$

$$\Rightarrow \frac{1}{2\sqrt{y_1} (1+y_2^2)^2}$$

$$I = \frac{y_2^2}{2(1+y_2^2)^2}$$

$$\text{inverted image} = \frac{y_2^2}{(1+y_2^2)^2}$$

$$a) f(y, y_2) = \frac{1}{2\pi} e^{-\frac{y_1}{2}} \frac{y^2}{(1+y_2^2)^2}$$

$$b) f(\eta, \lambda) = \frac{\lambda^{\eta}}{\Gamma(\eta)} = \frac{\lambda^{\eta}}{\Gamma(\eta)}$$

$$y = \frac{x_1}{x_1 + x_2} \quad z = x_1 + x_2$$

$$x_1 = \frac{x_1}{z} = x_1 = yz$$

$$x_2 = z(1-y)$$

$$f_1 = \frac{\lambda^{\eta_1} x_1^{\eta_1-1} e^{-\lambda x_1}}{\Gamma(\eta_1)} = \frac{\lambda^{\eta_1} x_1^{\eta_1-1} e^{-\lambda x_1}}{\Gamma(\eta_1)}$$

$$f(x_1) = \frac{\lambda^{\eta_1} x_1^{\eta_1-1} e^{-\lambda x_1}}{\Gamma(\eta_1)}$$

$$f(x_2) = \frac{\lambda^{\eta_2} x_2^{\eta_2-1} e^{-\lambda x_2}}{\Gamma(\eta_2)}$$

$$x_1 = yz \quad x_2 = z(1-y)$$

$$m = z - zy$$

$$z - zy + yz$$

$\frac{dx_1}{dy}$	$\frac{dx_1}{dz}$	= z y	= z
$\frac{dx_2}{dy}$	$\frac{dx_2}{dz}$		
		$z(1-y)$	z

$$J = Z$$

$$= \frac{\lambda^{n_1} x_1^{n_1-1} e^{-x_1 \lambda}}{\Gamma(n_1)} \cdot \frac{\lambda^{n_2} x_2^{n_2-1} e^{-x_2 \lambda}}{\Gamma(n_2)} \cdot Z$$

$$\Rightarrow \frac{\lambda^{(n_1+n_2)} e^{-(x_1+x_2)\lambda}}{\Gamma(n_1) \Gamma(n_2)} \cdot x_1^{n_1-1} x_2^{n_2-1} \cdot Z$$

$$\Rightarrow \frac{\lambda^{(n_1+n_2)} e^{-Z\lambda}}{\Gamma(n_1) \Gamma(n_2)} \cdot x_1^{n_1-1} x_2^{n_2-1} \cdot Z$$

$$\frac{\lambda^{(n_1+n_2)} e^{-Z\lambda}}{\Gamma(n_1) \Gamma(n_2)} (yz)^{n_1-1} \cdot (z(1-y))^{n_2-1} \cdot Z$$

$$\Rightarrow \lambda^{(n_1+n_2)} e^{-Z\lambda} y^{n_1-1} z^{n_1-1} \cdot (z(1-y))^{n_2-1} \cdot Z$$

$$\Rightarrow \lambda^{(n_1+n_2)} e^{-Z\lambda} y^{n_1-1} z^{n_1-1} \cdot [z(1-y)]^{n_2-1} \cdot Z$$

$$\textcircled{1} f(u) = e^{-u}$$

$$x_1 = x_1 + x_2 + x_3$$

$$y_2 = \frac{x_1 + x_2}{y_1}$$

$$y_3 = \frac{x_1}{x_1 + x_2}$$

$$x_1 + x_2 = y_2 y_1$$

$$x_1 = y_2 y_1 y_3$$

$$y_1 - y_2 y_1 = x_3$$

$$x_2 = \frac{y_2 y_1}{y_2} - y_2 y_1 y_3$$

$$= y_2 y_1 (1 - y_3)$$

$$\frac{dx_1}{dy_1}$$

$$\frac{dx_1}{dy_2}$$

$$\frac{dx_1}{dy_3}$$

$$\frac{dx_2}{dy_1}$$

$$\frac{dx_2}{dy_2}$$

$$\frac{dx_2}{dy_3}$$

$$\frac{dx_3}{dy_1}$$

$$\frac{dx_3}{dy_2}$$

$$\frac{dx_3}{dy_3}$$

$$\Rightarrow \begin{vmatrix} y_2 y_3 & y_1 y_3 & y_2 y_1 \\ y_2 - y_2 y_3 & y_1 - y_1 y_3 & -y_2 y_1 \\ 1 - y_2 & -y_1 & 0 \end{vmatrix}$$

$$\Rightarrow y_2 y_3 - y_2 y_1^2 - y_1 y_3 (y_2 y_1 - y_2^2 y_1)$$

$$+ y_2 y_1 (-y_2 y_1 + y_1 y_2 y_3 - y_1 + y_1 y_3 + y_1 y_2 y_3 - y_1 y_2 y_3)$$

$$y_2 y_3 - y_2 y_1^2 - y_1^2 y_3 y_2 + y_1^2 y_2^2 y_3$$

$$+ y_2 y_1^2 + y_1^2 y_2 y_3$$

$$\boxed{y_2 y_3 - y_2 y_1^2 + y_2 y_1^2 + y_1^2 y_2^2 y_3}$$

$$- y_1^2 y_3 - 2 y_2 y_1^2 + y_1^2 y_2^2 y_3$$

$$= -2 y_2 y_1^2$$

$$e^{-(y_1 + y_2 + y_3)}$$

$$f(y) \Rightarrow e^{-y_1} \cdot x | 2 y_2 y_1^2 |$$

$$\Rightarrow 2 y_2 y_1^2 e^{-y_1} \quad \text{or} \quad y_2 y_1^2 e^{-y_1}$$

$$\Rightarrow$$

$$f(y) = \int_0^1 y_2 y_1^2 e^{-y_1} dy_2$$

$$\Rightarrow y_1^2 e^{-y_1} \cdot \frac{y_2^2}{2} \Big|_0^1$$

$$\frac{y_1^2 e^{-y_1}}{2}$$

$$f(y) = \int_0^\infty y_1^2 e^{-y_1} dy_1$$

$$\int_0^{\infty} y_1^2 e^{-y_1} dy_1 \Rightarrow y_1^2 (-e^{-y_1}) - \int -e^{-y_1} \cdot 2y_1$$

$$-y_1 e^{-y_1} + 2 \int e^{-y_1} dy_1$$

$$\Rightarrow -y_1 e^{-y_1} - \int e^{-y_1}$$

$$\Rightarrow -y_1 e^{-y_1} + e^{-y_1}$$

$$-y_1 e^{-y_1} - 2y_1 e^{-y_1} + 2e^{-y_1}$$

$$= -3y_1 e^{-y_1} + 2e^{-y_1} \Big|_0^{\infty}$$

$$\Rightarrow 0 + 2y_2 = 2y_2$$

mult

$$\frac{y_1^2 e^{-y_1}}{2} \cdot 2y_2 = y_2 y_1^2 e^{-y_1} \text{ indep.}$$

since $\int_0^{\infty} \int_0^{\infty} f(y_1, y_2) dy_3 = 1$

②

$$x^2 - 2xy + y^2 + ny$$

$$(x - y)^2 + ny$$

$$x_1 = \ln n_1 \sim N(4, 1)$$

$$x_2 = \ln n_2 \sim N(3, 1)$$

$$x_3 = \ln n_3 \sim N(3, 0.5)$$

08

$$x_1 + x_2 + x_3 = \ln(n_1 n_2 n_3)$$

$$y_1 + y_2 + y_3 = \ln(n_1 n_2 n_3)$$

$$\ln w = 2 \ln x_1^2$$

$$w = e^2 x_1^2 x_2^{1.5} x_3^{1.28}$$

$$\ln w = 2 + 2 \ln x_1^2 + 1.5 \ln x_2 + 1.28 \ln x_3$$

$$\Rightarrow 2 + 2 \ln x_1^2 + 1.5 \ln x_2 + 1.28 \ln x_3$$

$$E(\ln w) = 2 + 2E(\ln x_1^2) + 1.5E(\ln x_2) + 1.28E(\ln x_3)$$

$$E(\ln w) = 2 + 2 \times 4 + 1.5 \times 3 + 1.28 \times 2$$

$$E(\ln w) = 17.06$$

$$\text{Var}(\ln w) = 0 + 4\sigma^2(\ln x_1) + 2.25\sigma^2(\ln x_2) + 1.28^2 \sigma^2(\ln x_3)$$

$$= 4 \times 1 + 2.25 \times 1 + 1.28^2 \times 0.5$$

$$= 7.0692$$

$$P(L < w < R) = 0.9$$

$$\Rightarrow \frac{4 - \ln x}{\sigma} = 0.90$$

$$\frac{4 - \ln x}{\sigma} = 0.95$$

$$\Rightarrow \frac{4 - \ln x}{\sigma} = 1.66$$

$$\Rightarrow \frac{17.06 - \ln x}{7.069} = 1.66$$

$$\ln x = 5.325 = 205.40$$

$$\text{Median} = e^4 = e^{17.06}$$

$$\text{Q8} \quad \frac{1}{2(1-p^2)} \left(\frac{n_1 - \mu_1}{\sigma_1} \right)^2 - 2 \left(\frac{n_1 - \mu_1}{\sigma_1} \right) \left(\frac{n_2 - \mu_2}{\sigma_2} \right) + \left(\frac{n_2 - \mu_2}{\sigma_2} \right)^2$$

$$\frac{1}{2(1-p^2)} = \frac{2}{3}$$

$$\frac{3}{4} = 1 - p^2 \quad p = \frac{1}{2}$$

$$\mu_1 = 0 \quad \sigma = 1$$

$$p = \frac{1}{2} \quad P(-1 < X < 1) \mid Y = 1$$

$$\mu_n = \mu + \frac{1}{2} \left(\frac{\sigma_n}{\sigma_y} \right) (y - 1)$$

$$\mu_1 = \frac{1}{2}, \quad \sigma = \frac{1}{2} \sqrt{1 - p^2}$$

$$\Rightarrow 1 / (1 - \frac{1}{4}) = \frac{4}{3}$$

$$\frac{-1n-4}{0} \quad \frac{-1-4}{0} \quad \frac{0-1-4}{0}$$

$$\Rightarrow \left(\frac{1-\frac{1}{2}}{\sqrt{3/2}} \right) + \phi \left(\frac{1-\frac{1}{2}}{\sqrt{3/2}} \right)$$

$$\Rightarrow \phi \left(\frac{1}{2} \times \frac{2}{\sqrt{3}} \right) - \phi \left(-\frac{1}{2} \times \frac{2}{\sqrt{3}} \right)$$

$$\phi \left(\frac{2}{\sqrt{3}} \right) - \phi \left(-\frac{2}{\sqrt{3}} \right)$$

$$\phi \left(\frac{1}{\sqrt{3}} \right) - \phi \left(-\frac{1}{\sqrt{3}} \right) = 2\phi(0.577) - 1$$

$$\Rightarrow 2 \times 0.7190 - 1$$

$$-\frac{3}{2} \times \frac{2}{\sqrt{3}} (-\sqrt{3})$$

$$-\frac{1}{2} \times \frac{2}{\sqrt{3}}$$

$$4.00x + 9.00y$$

$$\Rightarrow 1.3x + 9.3y = \frac{27}{4} + 3 =$$

$$\left(\frac{27}{4} + \frac{12}{4} + \frac{39}{4} \right)$$

$$\mu_A = 20 \quad \sigma_A^2 = 0.03$$

$$\mu_B = 14 \quad \sigma_B^2 = 0.01$$

$$33.6 < A+B < 34.4$$

$$\Rightarrow \frac{33.6-34}{0.02}$$

$$\Rightarrow \mu(A+B) = 34$$

$$\sigma_B^2 = 0.04$$

$$34.4-34$$

$$0.02$$

$$\phi(2) - \phi(-2)$$

$$2\phi(2) - 1 = 0.95 - 0.02 = 0.93$$

$$2 \times \frac{0.4}{\sqrt{2}}$$

$$E(X|Y=y) = \frac{\int_0^{\infty} x \cdot f(x,y) dx}{\int_0^{\infty} f(x,y) dx}$$

$$E(X) = \int_0^{\infty} x \cdot f(x,y) dx$$

$$E(X|Y=y)$$

$$\Rightarrow \int_0^2 x \cdot \frac{f(x,y)}{f(y)} dx$$

$$\text{Q3 } f(x,y) = \frac{1}{8} (6-x-y) \quad \begin{matrix} 0 < x < 2 \\ 2 < y < 4 \end{matrix}$$

$$f_y(x,y) = \int_2^4 \frac{1}{8} (6-x-y) dy$$

$$\Rightarrow \frac{1}{8} \left[6y - xy - \frac{y^2}{2} \right]_2^4$$

$$\Rightarrow \frac{1}{8} [24 - 4x - 8 - (12 - 2x - 2)]$$

$$\Rightarrow \frac{1}{8} [24 - 4x - 8 - 12 + 2x + 2]$$

$$\Rightarrow \frac{1}{8} [6 - 2x] = \frac{3-x}{4}$$

$$f_y(x,y) = \int_0^2 \frac{1}{8} (6-x-y) dx$$

$$\Rightarrow \frac{1}{8} \left[6x - \frac{x^2}{2} - yx \right]_0^2 = \frac{5-y}{4}$$

$$\frac{1}{8} [12 - 2 - 2y] = \frac{10-2y}{8}$$

$$E(Y/X=n) = \int_2^4 \frac{y \cdot f(\ln y)}{f_n} dy$$

$$\Rightarrow \int_2^4 \frac{y \cdot \frac{1}{8}(6-n-y)}{\frac{3-n}{4}} dy$$

$$\Rightarrow \int_2^4 \frac{y(6-n-y)}{2(3-n)} dy \Rightarrow \frac{1}{2(3-n)} \int_2^4 y(6-n-y) dy$$

$$\Rightarrow \frac{1}{2(3-n)} \left[\frac{6y^2}{2} - \frac{ny^2}{2} - \frac{y^3}{3} \right]_2^4$$

$$\Rightarrow \frac{1}{2(3-n)} \left[3 \times 16 - 8n - \frac{64}{3} - \left(12 - 2n - \frac{8}{3} \right) \right]$$

$$\Rightarrow \frac{1}{2(3-n)} \left[48 - 8n - \frac{64}{3} - 12 + 2n + \frac{8}{3} \right]$$

$$\frac{1}{2(3-n)} \left[36 - 6n + \frac{56}{3} \right]$$

$$\frac{1}{2(3-n)} \left[60 - 6n \right] = \frac{6[10-n]}{2(3-n)}$$

$$\frac{5-y}{4}$$

$$\frac{2y}{3}$$

$$\text{Cov}(xy) = E(xy) - E(x) \cdot E(y)$$

$$\int_2^4 \int_0^2 xy \cdot \frac{1}{8} (6-x-y) dx dy$$

$$\frac{1}{8} \int_2^4 dy \int_0^2 (6xy - x^2y - xy^2) dx$$

$$\frac{1}{8} \int_2^4 dy \left[3x^2y - \frac{x^3y}{3} - \frac{x^2y^2}{2} \right]_0^2$$

$$\frac{1}{8} \int_2^4 dy \left[12y - \frac{8y}{3} - \frac{4y^2}{2} \right]$$

$$\frac{1}{8} \int_2^4 dy \left[\frac{28y}{3} - \frac{4y^2}{2} \right]$$

$$\frac{1}{8} \int_2^4 \left[\frac{28y}{3} - 2y^2 \right] dy \Rightarrow \frac{1}{8} \left[\frac{28y^2}{6} - \frac{2y^3}{3} \right]$$

$$\frac{1}{8} \left[\frac{28 \cdot 4}{6} - \frac{54}{3} - \left(\frac{28 \cdot 2}{6} - \frac{2 \cdot 8}{3} \right) \right]$$

$$\frac{1}{8} \left[\frac{126}{3} - \frac{54}{3} - \frac{56}{3} + \frac{24}{3} \right]$$

$$\mu_1 = 28.4, \mu_2 = 31.6, \sigma_1 = 46.24$$

$$\sigma_2 = 54.76, \rho_{12} = 0.82$$

$$X, Y \sim \text{bivariate normal}(\mu_1, \sigma_1, \mu_2, \sigma_2, \rho_{12})$$

$$P(X > 30)$$

$$X = 28.4 + \frac{46.24}{54.76} \cdot (Y - 31.6) \rightarrow \text{when } Y = 30$$

$$P\left(Z > \frac{X - \mu_1}{\sigma_1}\right) = 1 - P\left(Z < \frac{X - \mu_1}{\sigma_1}\right)$$

$$\Rightarrow 1 - P\left(Z < \frac{30 - 28.4}{\sigma_1}\right)$$

$$P(Y > 35 | X = 30)$$

$$Y = \mu_2 + \left(\frac{\sigma_2}{\sigma_1}\right) \cdot \rho_{12} \cdot (X - \mu_1) + \sqrt{1 - \rho_{12}^2} \cdot \sigma_2 \cdot Z$$

35-

$$= \frac{28.4}{3} \cdot 3$$

$$= \frac{2.8}{5}$$

Q7

$$f(x, y) = \frac{1}{y} e^{-(y+\frac{x}{y})} \quad x > 0, y > 0$$

$$f_y(x, y) = \int_0^{\infty} \frac{1}{y} e^{-(y+\frac{x}{y})} dx$$

$$\Rightarrow \frac{1}{y} \left[\int_0^{\infty} e^{-y} \cdot e^{-\frac{x}{y}} dx \right]$$

$$\frac{1}{y} e^{-y} \cdot \int_0^{\infty} e^{-x/y} dx$$

$$\Rightarrow \frac{1}{y} e^{-y} \times \left. -y e^{-x/y} \right|_0^{\infty}$$

$$\frac{1}{y} e^{-y} \cdot y \cdot e^{-x/y} \Big|_0^{\infty} \Rightarrow e^{-y}$$

$$\frac{f(x, y)}{f_y(x, y)} = \frac{\frac{1}{y} e^{-(y+\frac{x}{y})}}{e^{-y}} = \frac{1}{y} e^{-x/y}$$

$$E(y) = \int_0^{\infty} y f_y(x, y) dy$$

$$\Rightarrow \int_0^{\infty} y e^{-y} dy$$

$$\Rightarrow -y e^{-y} - \int -e^{-y} dy$$

$$\Rightarrow -y e^{-y} - e^{-y} \Big|_0^{\infty} = +1 \text{ A.}$$

$$E(n) = E(E(n/y)) = E(n)$$

$$E[n/y] \Rightarrow f(n/y) = \frac{f_n(n, y)}{f_y(y)}$$

$$\Rightarrow \frac{\frac{1}{y} e^{-(y+n/y)}}{e^{-y}} = \frac{1}{y} e^{-n/y}$$

$$E(n/y)$$

First we need to find conditional Probability of n given y . then the expectat.

$$E(n/y) = \int_0^{\infty} n \frac{1}{y} e^{-n/y} dn$$

$$\Rightarrow \frac{1}{y} \int_0^{\infty} n e^{-n/y} dn$$

$$\Rightarrow \frac{1}{y} \left[-y e^{-n/y} \right]_0^{\infty} - \int_0^{\infty} -y e^{-n/y}$$

$$\frac{1}{y} \left[-y e^{-n/y} - y^2 e^{-n/y} \right]_0^{\infty} \quad E(n)=1$$

$$\Rightarrow -e^{-n/y} - y e^{-n/y} \Big|_0^{\infty}$$

$$\Rightarrow 0 - 0 - (-1 - 0)$$

$$\Rightarrow 1$$

$$E(y) = E(E(y|x))$$

$$\text{var}(y) = E(y^2) - (E(y))^2$$

$$E(y^2) = \int_0^{\infty} y^2 e^{-y} dy$$

$$\Rightarrow -y^2 e^{-y} - \int [2y - e^{-y}]$$

$$= -y^2 e^{-y} - 2 \int y e^{-y} dy$$

$$\Rightarrow -y^2 e^{-y} - 2 \left[-y e^{-y} - \int -e^{-y} \right]$$

$$= -y^2 e^{-y} - 2 \left[-y e^{-y} - e^{-y} \right]_0^{\infty}$$

$$\Rightarrow -y^2 e^{-y} + 2y e^{-y} + 2e^{-y} \Big|_0^{\infty}$$

$$\Rightarrow 2$$

$$\text{var}(y) = 2 - 1 = 1$$

$$E(n^2/y) = x^2$$

$$x < 3/4 / y = 1/6$$

$$E(y/x=1/6)$$

$$\textcircled{9} \quad f(n,y) = \frac{1}{4}(1+ny) \quad |n| < 1; |y| < 1$$

$$u = x^2, \quad v = y^2$$

$$x = \pm \sqrt{u}$$

$$y = \pm \sqrt{v}$$

$$\frac{dn}{du} = \frac{1}{2\sqrt{u}}$$

$$\frac{dn}{dv} = 0$$

$$\frac{dy}{dv} = \frac{1}{2\sqrt{v}}$$

$$\frac{1}{4\sqrt{uv}}$$

$$f(n,y) \Rightarrow \left(\frac{1+ny}{4} \right) \cdot \frac{1}{4\sqrt{uv}} \Rightarrow \frac{1}{2\sqrt{uv}} (1 + \sqrt{uv})$$

$0 < u < 1$
 $0 < v < 1$

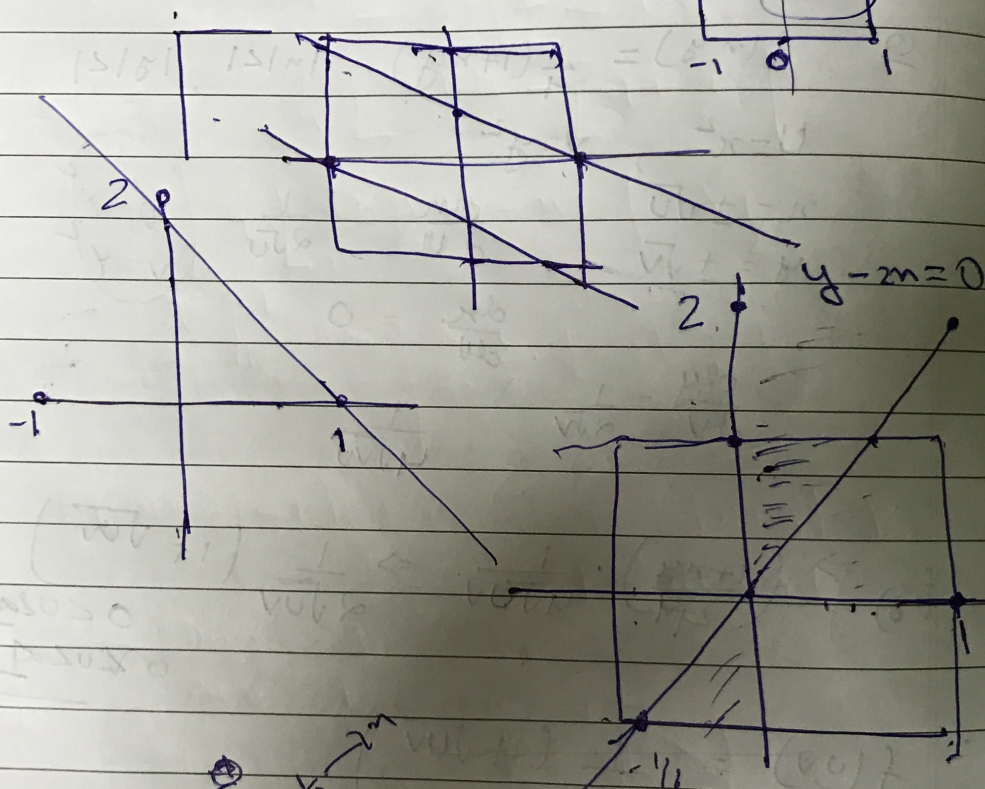
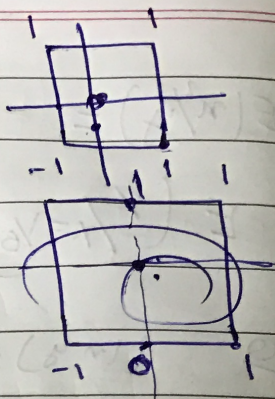
$$f(u,v) = \frac{1}{2\sqrt{uv}} (1 + \sqrt{uv})$$

$$\frac{1 + \sqrt{uv}}{2\sqrt{uv}} \Rightarrow \frac{1}{2\sqrt{uv}} + \frac{1}{2}$$

$$f(n,y) = \frac{1}{4}(1+ny)$$

$$f(x, y) = \frac{1}{4}(1+xy)$$

$$P(2 \leq y)$$



$$2 \times \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} \frac{1}{4}(1+xy) dx dy$$

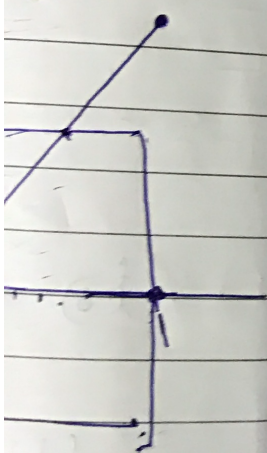
$$\frac{1}{8} + \frac{1}{64} = \frac{9}{64}$$

$$\frac{9}{32}$$

$$\frac{8 + y^2}{8 \cdot 64} \Big|_0^{\frac{1}{2}} \Rightarrow \frac{1}{4} \left(\frac{1}{2} + \frac{y}{8} \right) \Rightarrow \frac{1}{8} + \frac{y}{32}$$



$$y - m = 0$$



$$= \frac{9}{64}$$

2

or

$$\int_{-1/2}^1 \int_{-1/2}^{1/2} \frac{1+m}{4} + \int_{1/2}^1 \frac{m}{4}$$

$$\Rightarrow \frac{1}{4} \int_{-1/2}^{1/2} \left(y + \frac{my^2}{2} \right)_{-1}^{2n}$$

$$\frac{1}{4} \left[2n + 2n^3 - \left(-1 + \frac{n}{2} \right) \right]$$

$$\Rightarrow \frac{1}{4} \left[2n + 2n^3 + 1 - \frac{n}{2} \right] = \frac{1}{4} \left[\frac{3n}{2} + 2n^3 + 1 \right]$$

$$\int_{-1/2}^{1/2} \frac{1}{4} \left[\frac{3n}{2} + 2n^3 + 1 \right]$$

$$\frac{7}{4} \left[\frac{3n^2}{4} + \frac{2ny}{4} + n \right]_{-1/2}^{1/2}$$

$$\frac{1}{4} \left[1 \right] = \frac{1}{4}$$

$$\Rightarrow \frac{1+m}{4} \frac{dn}{4}$$

$$\frac{1}{4} \left[n + \frac{n^2}{2} \right]_{1/2}^1$$

$$\frac{1}{4} + \frac{7}{32}$$

$$= \frac{8+7}{32}$$

$$\Rightarrow \frac{15}{32}$$

$$\Rightarrow \frac{1}{4} \left[1 + \frac{1}{2} - \frac{1}{2} - \frac{1}{8} \right]$$

$$\Rightarrow \frac{1}{4} \left[\frac{3}{8} \right] = \frac{3}{32}$$

$$\int_{1/2}^1 \int_{-1}^1 \frac{1+ny}{4} dy dn$$

$$\Rightarrow \frac{1}{4} \left[y + \frac{ny^2}{2} \right]_{-1}^1$$

$$\frac{1}{4} \left[1 + \frac{n}{2} - \left[-1 + \frac{n}{2} \right] \right]$$

$$\frac{1}{4} \left[1 + \frac{n}{2} + 1 - \frac{n}{2} \right] \Rightarrow \frac{1}{2}$$

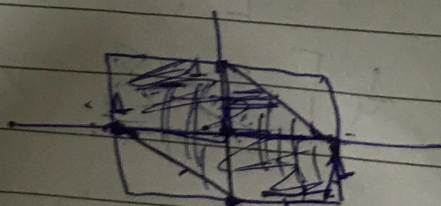
$$\frac{n}{2} \Big|_{1/2}^1 \Rightarrow \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(|n+y| < 1)$$

$$|n+y| = 1$$

$$-1 < n+y < 1$$

$$\begin{aligned} n+y &= -1 \\ 1-n &= y+y \\ n+y &= 1 \end{aligned}$$



$$\int_0^1 \int_{-1}^1 \frac{1+ny}{4} dy dn$$



$$\int_0^1 \int_{-1}^1 \frac{1+ny}{4} dy dn$$

$$+ \int_0^1 \int_{-1}^1 \frac{1+ny}{4} dy dn + \int_0^1 \int_{-1}^1 \frac{1+ny}{4} dy dn$$

$$-(1+y)$$

$$\Rightarrow \frac{1}{4} \left[x + \frac{x^2}{2} \right]_{-y}^{1-y}$$

~~$$\frac{1}{4} \left[1-y + \frac{(1-y)^2}{2} \right] - \frac{1}{4} \left[-y + \frac{y^2 + 1 - 2y}{2} \right]$$~~

$$\frac{1}{4} \left[x + \frac{x^2}{2} \right]_{-y}^{1-y}$$

$$\frac{1}{4} \left[1-y + \frac{(1-y)^2}{2} \right]$$

$$\frac{1}{4} \left[y - \frac{y^2}{2} + \frac{1}{2} \left[\frac{y^4}{4} - \frac{2y^3}{3} + \frac{y^2}{2} \right] \right]_{-1}^1$$

$$\frac{1}{4} \left[1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{3} + \frac{1}{2} \right]$$

$$\frac{1}{4} \left[\frac{1}{4} + \frac{2}{8} - \frac{1}{3} + \frac{1}{2} \right] = \frac{19}{96}$$

$$\frac{1}{4} \left[x + \frac{x^2}{2} \right]_{-(1+y)}^{-(1+y)}$$

$$\frac{1}{4} \left[-(1+y) + \frac{(1+y)^2}{2} \right]$$

~~$$\frac{1}{4} \left[-y - \frac{y^2}{2} + \frac{1}{2} \left[y^3 + 2y^2 + y \right] \right]$$~~

$$\frac{1}{4} \left[-y - \frac{y^2}{2} + \frac{y^4}{8} + \frac{y^3}{3} + \frac{y^2}{4} \right]_{-1}^1$$

$-(1+y)$

$$\frac{19}{96} - \frac{17}{96}$$

$$\int_0^1 \int_0^1 \left(1 + \frac{xy}{4}\right) dx dy$$

$$\int_0^1 \left. \frac{x + \frac{x^2 y}{4}}{1} \right|_0^1 dy \Rightarrow \int_0^1 \frac{-1 + y}{4} dy$$

$$\left. -\frac{y}{4} + \frac{y^2}{8} \right|_0^1 \Rightarrow \left(-\frac{1}{4} + \frac{1}{8} \right) \frac{1}{4}$$

$$\Rightarrow -\frac{1}{8}$$

$$\int_0^1 \int_0^1 \frac{x + \frac{x^2 y}{4}}{4} dx dy$$

$$\Rightarrow \int_0^1 \left[\frac{1 + y}{4} \right] dy \Rightarrow \left. \frac{y + \frac{y^2}{2}}{4} \right|_0^1$$

$$\Rightarrow -1 + \frac{1}{2} \Rightarrow -\frac{1}{2}$$

$$\frac{1}{2} + \frac{17}{24} + \frac{96}{96} = \frac{48 + 68 + 96}{96}$$

$$\frac{19}{96} - \frac{17}{24} + \frac{1}{4} = \frac{96 - 68 + 24}{96} \Rightarrow \frac{120 - 68}{96}$$

$$\Rightarrow \frac{52}{96} = \frac{13}{24}$$